

Propellant-Remaining Modeling

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ABSTRACT

A successful satellite mission is predicated upon the proper maintenance of the spacecraft's orbit and attitude. One requirement for planning and predicting the orbit and attitude is the accurate estimation of the propellant remaining onboard the spacecraft. For geosynchronous satellites, a precise propellant-remaining estimation is of particular importance. Twenty kilograms (kg) of propellant can add a year to the operational lifetime of a satellite such as the Geostationary Operational Environment Satellite (GOES)-I. Moreover, the geosynchronous ring is becoming cluttered with propellant-depleted satellites; therefore, an extra 3 to 4 kg of fuel may be required to deorbit an expiring satellite out of the geosynchronous ring. For GOES-I, which is loaded with over 670 kg of oxidizer and over 420 kg of fuel, accounting for 20 kg of propellant requires accuracy in propellant-remaining estimation of within 2 percent. Budgeting for the 3 kg of propellant at the end of the mission requires a method with an accuracy of within 0.5 percent.

This paper focuses on the three methods that were developed for calculating the propellant budget: in particular, the errors associated with each method and the uncertainties in the variables required to determine the propellant remaining that contribute to these errors. Based on these findings, a strategy will be developed for improved propellant-remaining estimation. The first method is based on Boyle's law, which relates the values of pressure, volume, and temperature (PVT) of an ideal gas. The PVT method is used for both the monopropellant and the bipropellant engines. The second method is based on the engine performance tests, which provide data that relate thrust and specific impulse (I_{sp}) associated with a propellant tank to that tank's pressure. Two curves representing thrust and specific impulse as functions of pressure are then generated using a polynomial fit on the engine performance data. The third method involves a computer simulation of the propellant system. The propellant flow is modeled by creating a conceptual model of the propulsion system configuration, taking into account such factors as the propellant and pressurant tank characteristics, thruster functionality, and piping layout.

Finally, this paper presents a thrust calibration technique that uses differential correction with the computer simulation method of propellant-remaining modeling. Thrust calibration will provide a better assessment of thruster performance and therefore enable a more accurate estimation of propellant consumed during a given maneuver.

1. METHODS OF COMPUTING PROPELLANT REMAINING

Introduction

Before presenting the detailed descriptions of the propellant estimation methods, a brief introduction on the propulsion system operation during a spacecraft mission is in order.

For the liquid propellant/gas pressurant system considered in this paper two modes of operations are feasible: blowdown and pressure regulated. In blowdown mode, the propellant tank is pressurized by the gas pressurant and then during the mission the propellant tank pressure is allowed to decay as propellant is "blown out" of the tank when the thrusters are firing. In pressure regulated mode, the propellant tank pressure is maintained constant by supplying additional pressurant gas into the propellant tank during the thruster firing. The advantage of the pressure regulated mode is the constant propellant flow rate which is necessary to maintain in a bipropellant type engine to ensure constant mixture ratio for optimum thruster performance (see Section I, Method 1).

The propellant estimation methods described here can be used to model an engine operating in both blowdown and pressure regulated mode, blowdown mode only, and pressure regulated mode only (Methods 1, 2, and 3 respectively).

Two phases of the mission are mentioned in this paper: the transfer orbit phase—when the satellite is maneuvered to achieve mission orbit; and the station keeping phase—when the satellite is maneuvered to maintain the mission orbit. For GOES-I, 86 percent of the propellant is used during the transfer orbit phase (NASA phase). GRO on the other hand is inserted into the mission orbit by the launch vehicle; therefore, ideally, 100 percent of propellant is used for station keeping and controlled reentry.

Method 1: Pressure, Volume, Temperature

The PVT method is based on an assumption of ideal gas behavior of the pressurant gas. Since the pressurant gas is helium, the ideal gas approximation is valid. Boyle's law is then used to estimate the propellant remaining based on the amount of pressurant that was forced into the propellant tank. The procedure is as follows:

Suppose that the volume of propellant displaced from the tank is equal to the volume of pressurant forced into the tank, given that pressurant and propellant do not mix and that the tank volume does not change. Then,

$$dM_f = \rho_{of} \cdot dV_{He,f} \quad (1-1)$$

where dM_f = propellant forced out of the tank
 ρ_{of} = propellant density
 $dV_{He,f}$ = volume of pressurant forced into the tank

Using Boyle's law to compute the pressurant volume and writing the propellant density as a function of tank temperature, we have

$$dV_{He,f} = \frac{R \cdot dM_{He} \cdot Z_{He} \cdot T_{He}}{P_{He}} \quad (1-2)$$

where R = pressurant (helium) gas constant
 dm_{He} = mass of pressurant forced into the propellant tank
 Z_{He} = pressurant compressibility
 T_{He} = pressurant temperature
 P_{He} = pressurant pressure

$$\rho_f = b_0 + b_1 \cdot T + b_2 \cdot T^2 \quad (1-3)$$

here T = propellant tank temperature
 b_0, b_1, b_2 = propellant density coefficients

since pressurant and propellant are in the same tank and do not mix,

$$T_{He} = T \quad (1-4)$$

$$P_{He} = P - P_{sat} \quad (1-5)$$

here P = propellant tank pressure
 P_{sat} = propellant saturation pressure

propellant saturation pressure may be expressed as a function of temperature, T ,

$$P_{sat} = 10^{(a_0 - a_1/T - a_2/T^2)} \quad (1-6)$$

here a_0, a_1, a_2 = propellant saturation pressure coefficients

pressurant compressibility may be expressed as a function of pressurant pressure, P_{He} , and temperature, T_{He} ,

$$Z_{He} = 1 + P_{He} \cdot [\beta + \Gamma \cdot (T - T_s)] \quad (1-7)$$

here β, Γ = pressurant compressibility coefficients
 T_s = pressurant standard temperature

The expression for propellant used may then be rewritten as a function of tank pressure and temperature and the mass of pressurant forced into the propellant tank,

$$dM_f = R \cdot dM_{He} \cdot [b_0 + b_1 \cdot T + b_2 \cdot T^2] \cdot \left[\frac{T}{P - 10^{(a_0 - a_1/T - a_2/T^2)}} + \beta \cdot T + \Gamma \cdot T^2 - \Gamma \cdot T_s \cdot T \right] \quad (1-8)$$

The accuracy of the propellant estimation using this method is only as good as the certainty in the values of P, T , and dM_{He} . The uncertainty in the tank pressure and temperature is based on the ratings of the pressure transducers and the temperature sensors, as well as the telemetry signal resolution. The uncertainty in the pressurant mass forced into the propellant tank can be considered at most as great as the uncertainty in the pressurant mass leaving the pressurant tank (assuming there are no leaks), which is a function of the loading conditions and the telemetry readings of the pressure and temperature of the pressurant tank.

Since pressurant gas behaves as an ideal gas, Boyle's law applies as follows:

$$\frac{P_o \cdot V_o}{m_o \cdot Z_{He,o} \cdot T_o} = \frac{P_{He} \cdot V_{He}}{m_{He} \cdot Z_{He} \cdot T_{He}} \quad (1-9)$$

where P_o = loading pressurant tank pressure
 T_o = loading pressurant tank temperature
 V_o = loading pressurant tank volume
 m_o = loading pressurant mass
 $Z_{He,o}$ = loading pressurant compressibility
 P_{He} = pressurant tank pressure
 T_{He} = pressurant tank temperature
 V_{He} = pressurant tank volume
 m_{He} = pressurant mass
 Z_{He} = pressurant compressibility

Using the fact that pressurant tank volume is a function of pressurant tank pressure and temperature,

$$V_o = V_h + q_1 \cdot (P_o - P_h) + q_2 \cdot (T_o - T_h) \quad (1-10)$$

$$V_{He} = V_h + q_1 \cdot (P_{He} - P_h) + q_2 \cdot (T_{He} - T_h) \quad (1-11)$$

where V_h = standard pressurant tank volume
 P_h = standard pressurant tank pressure
 T_h = standard pressurant tank temperature
 q_1 = pressure coefficient
 q_2 = temperature coefficient

and the fact that compressibility is also a function of tank pressure and temperature, Equation (1-7), the amount of pressurant forced into the propellant tank may be expressed as follows:

$$dM_{He} = m_o - m_{He} \quad (1-12)$$

or

$$dM_{He} = m_o \cdot \left\{ 1 - \frac{P_{He} \cdot T_o}{P_o \cdot T_{He}} \cdot \frac{V_h + q_1 \cdot (P_{He} - P_h) + q_2 \cdot (T_{He} - T_h)}{V_h + q_1 \cdot (P_o - P_h) + q_2 \cdot (T_o - T_h)} \right. \\ \left. \cdot \frac{1 + P_o \cdot [\beta + \Gamma \cdot (T_o - T_s)]}{1 + P_{He} \cdot [\beta + \Gamma \cdot (T_{He} - T_s)]} \right\} \quad (1-13)$$

The error in propellant estimation may be expressed in terms of the uncertainties in propellant tank temperatures and pressures obtained from telemetry and the uncertainty in the pressurant mass forced into the propellant tank. Therefore, the error in propellant estimation due to these variables can be defined in standard fashion as

$$\text{Error}_P = \frac{\delta (dM_f)}{\delta (P)} \cdot P_{\text{uncertainty}} \quad (1-14a)$$

$$\text{Error}_T = \frac{\delta (dM_f)}{\delta (T)} \cdot T_{\text{uncertainty}} \quad (1-14b)$$

$$\text{Error}_{dM_{He}} = \frac{\delta (dM_f)}{\delta (dM_{He})} \cdot dM_{He_{uncertainty}} \quad (1-14c)$$

The uncertainty in pressurant mass can be expressed as 1σ error in the pressurant mass due to the uncertainties in the pressurant tank loading conditions and telemetry uncertainty in pressurant tank pressure and temperature:

$$dM_{He_{uncertainty}} = \sqrt{(\text{Error}_{P_{He}})^2 + (\text{Error}_{T_{He}})^2 + (\text{Error}_{P_o})^2 + (\text{Error}_{T_o})^2 + (\text{Error}_{m_o})^2} \quad (1-14d)$$

where

$$\text{Error}_{P_{He}} = \frac{\delta (dM_{He})}{\delta (P_{He})} \cdot P_{He_{uncertainty}} \quad (1-14e)$$

$$\text{Error}_{T_{He}} = \frac{\delta (dM_{He})}{\delta (T_{He})} \cdot T_{He_{uncertainty}} \quad (1-14f)$$

$$\text{Error}_{P_o} = \frac{\delta (dM_{He})}{\delta (P_o)} \cdot P_{o_{uncertainty}} \quad (1-14g)$$

$$\text{Error}_{T_o} = \frac{\delta (dM_{He})}{\delta (T_o)} \cdot T_{o_{uncertainty}} \quad (1-14h)$$

$$\text{Error}_{m_o} = \frac{\delta (dM_{He})}{\delta (m_o)} \cdot m_{o_{uncertainty}} \quad (1-14i)$$

Blowdown mode operation may be simulated by assuming that there is no change in pressurant mass in the propellant tank (i.e., pressurant tank is shut off). Then, the propellant remaining becomes a function of the propellant tank pressure and temperature change where $dM_{He} = \text{constant}$.

In a bipropellant propulsion system such as the one in GOES-I, the pressurant forced into a given tank is a function of the split ratio (the ratio of pressurant mass forced into the two tanks) as well as a function of the pressurant mass leaving the pressurant tank. The split ratio is, in turn, a function of the pressures and temperatures of the two propellant tanks and the mixture ratio (the ratio of the mass flow rates of the two propellants). The pressures and the temperatures are obtained from telemetry; the mixture ratio is defined by the manufacturer to ensure the optimum thruster performance. The expression for the split ratio in terms of the above quantities is derived as follows:

$$\text{Split Ratio} = \frac{dM_{He1}}{dM_{He2}} \quad (1-15)$$

where dM_{He1} = mass of pressurant forced into propellant tank 1
 dM_{He2} = mass of pressurant forced into propellant tank 2

Combining Equations (1-1) and (1-2) and introducing subscripts to distinguish between the two propellant tanks,

$$\frac{dM_{f1}}{dM_{He1}} = \rho_{o_{f1}} \cdot \frac{R \cdot Z_{He1} \cdot T_{He1}}{P_{He1}} \quad (1-16a)$$

and

$$\frac{dM_{f2}}{dM_{He2}} = \rho_{of2} \cdot \frac{R \cdot Z_{He2} \cdot T_{He2}}{P_{He2}} \quad (1-16b)$$

Then, using the definition of the mixture ratio (MR) and Equations (1-3) and (1-7) for propellant densities and pressurant compressibility, respectively,

$$\begin{aligned} \text{Split Ratio} = \text{MR} \cdot \frac{b_{f0} + b_{f1} \cdot T_2 + b_{f2} \cdot T_2^2}{b_{o0} + b_{o1} \cdot T_1 + b_{o2} \cdot T_1^2} \cdot \frac{P_{He1}}{P_{He2}} \cdot \frac{T_{He2}}{T_{He1}} \\ \cdot \frac{1 + P_{He2} \cdot [\beta + \Gamma \cdot (T_2 - T_s)]}{1 + P_{He1} \cdot [\beta + \Gamma \cdot (T_1 - T_s)]} \end{aligned} \quad (1-17)$$

where pressurant partial pressure can be expressed in terms of the pressure and temperature of the propellant tank using Equations (1-5) and (1-6). Then the pressurant forced into either propellant tank as a function of the total pressurant leaving the pressurant tank and the split ratio is

$$dM_{He1} = dM_{He} \cdot \frac{\text{Split Ratio}}{(\text{Split Ratio} + 1)} \quad (1-18a)$$

$$dM_{He2} = dM_{He} \cdot \frac{1}{(\text{Split Ratio} + 1)} \quad (1-18b)$$

Therefore, the set of error equations (Equation (1-14)) for a given propellant tank (1 or 2) must be expanded to include the errors in pressurant mass forced into the propellant tank due to the uncertainties in the mixture ratio and the uncertainties in pressure and temperature of the other propellant tank in the system. That is,

$$\text{Error}_{P1} = \frac{\delta(dM_{He})}{\delta(P_1)} \cdot P1_{\text{uncertainty}} \quad (1-19a)$$

$$\text{Error}_{T1} = \frac{\delta(dM_{He})}{\delta(T_1)} \cdot T1_{\text{uncertainty}} \quad (1-19b)$$

$$\text{Error}_{P2} = \frac{\delta(dM_{He})}{\delta(P_2)} \cdot P2_{\text{uncertainty}} \quad (1-19c)$$

$$\text{Error}_{T2} = \frac{\delta(dM_{He})}{\delta(T_2)} \cdot T2_{\text{uncertainty}} \quad (1-19d)$$

$$\text{Error}_{MR} = \frac{\delta(dM_{He})}{\delta(MR)} \cdot MR_{\text{uncertainty}} \quad (1-19e)$$

the 1σ error in pressurant forced into a given propellant tank then becomes

$$\begin{aligned} dM_{He_{uncertainty}} = & [(Error_{P_{He}})^2 + (Error_{T_{He}})^2 + (Error_{P_o})^2 + (Error_{T_o})^2 + (Error_{m_o})^2 \\ & + (Error_{P_1})^2 + (Error_{T_1})^2 + (Error_{P_2})^2 + (Error_{T_2})^2 + (Error_{MR})^2]^{1/2} \end{aligned} \quad (1-19f)$$

Using the data for the GOES-I propellant system operating in pressure regulated mode as an example: given nominal operating propellant tank pressures and temperatures (indexes 1 and 2 indicate the oxidizer and the fuel tanks, respectively) of

$$P_1 = P_2 = 230 \text{ psi} \quad T_1 = T_2 = 20^\circ \text{ C}$$

and pressurant tank loading mass, pressure, and temperature

$$P_o = 3300 \text{ psi} \quad T_o = 21^\circ \text{ C} \quad m_o = 2.54 \text{ lbm}$$

and assuming the operating pressurant tank temperature remains constant

$$T_{He} = T_o$$

the pressurant tank pressure at the end of NASA phase is reduced to

$$P_{He} = 200 \text{ psi}$$

and the optimum mixture ratio as supplied by the manufacturer is

$$MR = 1.610$$

Assuming that there are no uncertainties in loading conditions of the pressurant tank and mass, the partials are computed to be

$$\frac{\delta(dM_f)}{\delta(P_2)} = -1.4 \text{ lbm/psi} \quad \frac{\delta(dM_f)}{\delta(T_2)} = 0.57 \text{ lbm/}^\circ \text{ K} \quad \frac{\delta(dM_f)}{\delta(dM_{He2})} = 339.5$$

The partials to compute the uncertainty in pressurant mass forced into the fuel tank are

$$\begin{aligned} \frac{\delta(dM_{He2})}{\delta(P_{He})} &= -0.0004 \text{ lbm/psi} & \frac{\delta(dM_{He2})}{\delta(T_{He})} &= +0.003 \text{ lbm/}^\circ \text{ K} \\ \frac{\delta(dM_{He2})}{\delta(P_1)} &= -0.002 \text{ lbm/psi} & \frac{\delta(dM_{He2})}{\delta(T_1)} &= +0.002 \text{ lbm/}^\circ \text{ K} \\ \frac{\delta(dM_{He2})}{\delta(P_2)} &= +0.002 \text{ lbm/psi} & \frac{\delta(dM_{He2})}{\delta(T_2)} &= -0.001 \text{ lbm/}^\circ \text{ K} \\ \frac{\delta(dM_{He2})}{\delta(MR)} &= -0.3 \text{ lbm} \end{aligned}$$

The uncertainties in pressure and temperature due to the telemetry resolution are

$$\begin{aligned} P1_{\text{uncertainty(telem)}} &= \pm 0.4 \text{ psi} & T1_{\text{uncertainty(telem)}} &= \pm 1.0^\circ \text{ K} \\ P2_{\text{uncertainty(telem)}} &= \pm 0.4 \text{ psi} & T2_{\text{uncertainty(telem)}} &= \pm 1.0^\circ \text{ K} \\ PHe_{\text{uncertainty(telem)}} &= \pm 0.4 \text{ psi} & THe_{\text{uncertainty(telem)}} &= \pm 1.0^\circ \text{ K} \end{aligned}$$

The uncertainty in pressure due to the transducer accuracy and the resulting total root-mean-square uncertainty in pressure readings are

$$\begin{aligned} P1_{\text{uncertainty(trans)}} &= \pm 1.35 \text{ psi} & P1_{\text{uncertainty(tot)}} &= \pm 1.41 \text{ psi} \\ P2_{\text{uncertainty(trans)}} &= \pm 1.35 \text{ psi} & P2_{\text{uncertainty(tot)}} &= \pm 1.41 \text{ psi} \\ PHe_{\text{uncertainty(trans)}} &= \pm 1.35 \text{ psi} & PHe_{\text{uncertainty(tot)}} &= \pm 1.41 \text{ psi} \end{aligned}$$

The uncertainty in the mixture ratio as supplied by the manufacturer is

$$MR_{\text{uncertainty}} = \pm 0.024$$

Thus, the errors in fuel used associated with the resulting uncertainties in the fuel tank pressure, temperature, and amount of pressurant forced into the fuel tank (3σ error in pressurant forced into the fuel tank is ± 0.027 lbm) are

$$\text{Error}_{P2} = 2.0 \text{ lbm} \quad \text{Error}_{T2} = 0.57 \text{ lbm} \quad \text{Error}_{dMHe} = 9.17 \text{ lbm}$$

This shows that the fuel-used estimate for the GOES-I spacecraft in the pressure-regulated mode has a 3σ uncertainty of ± 28.2 lbm. Thus, given a 911.6 lbm estimated fuel usage, the relative error in fuel-remaining estimation is 3.1 percent.

Method 2: Thrust and Specific Impulse Performance Data

This section presents the mathematical argument for the thrust and specific impulse (I_{sp}) curves method of computing the propellant consumed from a tank during a specified time interval. These curves describe thrust and I_{sp} as functions of pressure and temperature. The method assumes that the propellant system behaves according to the thrust and I_{sp} performance curves derived through empirical testing of the propellant system. The equations to describe these curves are derived through polynomial fitting and take on the following form when the first three terms of the polynomial are used:

$$F = c_0 + c_1 \cdot P - c_2 \cdot P^2 \cdot \left[\frac{T}{T_{\text{ref}}} \right]^{c_3 + c_4 \cdot P} \quad (1-20)$$

$$I_{sp} = d_0 + d_1 \cdot P - d_2 \cdot P^2 \cdot \left[\frac{T}{T_{\text{ref}}} \right]^{d_3 + d_4 \cdot P} \quad (1-21)$$

where F = thrust
 I_{sp} = specific impulse
 P = propellant tank pressure
 T = inlet propellant temperature
 T_{ref} = inlet propellant temperature at which the data were taken
 c_0, c_1, c_2, c_3, c_4 = thrust polynomial coefficients
 d_0, d_1, d_2, d_3, d_4 = I_{sp} polynomial coefficients

Note that if T and T_{ref} are equal, then thrust and I_{sp} are functions of tank pressure only.)

Given thrust and I_{sp} , the propellant flow rate is easily determined:

$$\dot{\omega} = F/I_{sp} \quad (1-22)$$

where $\dot{\omega}$ = propellant flow rate.

The propellant mass escaping the tank during a certain time period is

$$dM_f = \int_{t_0}^{t_1} \dot{\omega} \cdot dt \quad (1-23)$$

where dt = time period.

Substituting Equations (1-20) and (1-21) into Equation (1-22), integrating with respect to pressure and temperature, dividing by the change in pressure and temperature to obtain the average flow rate, and then substituting into Equation (1-23), we find that

$$dM_f = \int_{t_0}^{t_1} \frac{\int_{P_0}^{P_1} \frac{\dot{\omega} dT}{T_1 - T_0} dP}{P_1 - P_0} dt \quad (1-24)$$

where P_{t_0} = tank pressure at t_0
 P_{t_1} = tank pressure at t_1
 T_{t_0} = tank temperature at t_0
 T_{t_1} = tank temperature at t_1

Note that when there is no change in temperature or pressure in a given time interval, the flow rate is constant with respect to that variable over this time interval, and therefore, the integration step with respect to the unchanging variable should be omitted.

Since thrust and I_{sp} method depends on tank pressure variation, it is meaningful to use this method only in the blowdown mode of operation when a significant change in tank pressures can be observed.

The errors associated with this method are inherent to the instruments used in deriving thrust and I_{sp} data points, as well as the data regularity required to produce a close polynomial fit. Assuming that thrust and I_{sp} are well-behaved functions and that the instruments used to take the data are extremely accurate, the error in determining the propellant flow rate is then a function of uncertainties in the burn start and stop time and of the uncertainties in tank pressure and temperature at burn start and stop time.

$$\text{Error}_{P0} = \frac{\delta (dM_f)}{\delta (P0)} \cdot P0_{\text{uncertainty}} \quad (1-25a)$$

$$\text{Error}_{P1} = \frac{\delta (dM_f)}{\delta (P1)} \cdot P1_{\text{uncertainty}} \quad (1-25b)$$

$$\text{Error}_{T0} = \frac{\delta (dM_f)}{\delta (T0)} \cdot T0_{\text{uncertainty}} \quad (1-26a)$$

$$\text{Error}_{T1} = \frac{\delta (dM_f)}{\delta (T1)} \cdot T1_{\text{uncertainty}} \quad (1-26b)$$

$$\text{Error}_{t0} = \frac{\delta (dM_f)}{\delta (t0)} \cdot t0_{\text{uncertainty}} \quad (1-27a)$$

$$\text{Error}_{t1} = \frac{\delta (dM_f)}{\delta (t1)} \cdot t1_{\text{uncertainty}} \quad (1-27b)$$

The following example for the Gamma Ray Observatory (GRO) satellite, which operates in blowdown mode, shows the calculations of the fuel used from a tank given that tank's pressures at start and end of burn and the burn duration.

Using GRO main satellite thrusters performance coefficients and assuming that the tank temperature remains the same as the reference temperature during which the curves data were taken:

$$F = (3.3502 \text{ lbf}) + (0.39898 \text{ lbf/psi}) \cdot P - (0.0001463 \text{ lbf/psi}^2) \cdot P^2$$

$$I_{sp} = (222.52 \text{ s} \cdot g) + (0.064329 \text{ s} \cdot g) \cdot P - (0.0000672 \text{ s} \cdot g) \cdot P^2$$

where the units of thrust and I_{sp} coefficients are as appropriate, and, given a 2-minute ascent maneuver and propellant tank pressures at the start and end of the maneuver of

$$dt = 120 \text{ sec}$$

$$P0 = 400 \text{ psi}$$

$$P1 = 334 \text{ psi}$$

the partials are computed to be

$$\frac{\delta(dM_f)}{\delta(P0)} = 0.072 \text{ lbm/psi}$$

$$\frac{\delta(dM_f)}{\delta(t0)} = 0.548 \text{ lbm/sec}$$

$$\frac{\delta(dM_f)}{\delta(P1)} = 0.071 \text{ lbm/psi}$$

$$\frac{\delta(dM_f)}{\delta(t1)} = -0.548 \text{ lbm/sec}$$

The uncertainties in pressure and time due to the telemetry resolution are

$$P_{0\text{uncertainty}} = \pm 3.0 \text{ psi} \quad t_{0\text{uncertainty}} = \pm 0.256 \text{ sec}$$

$$P_{1\text{uncertainty}} = \pm 3.0 \text{ psi} \quad t_{1\text{uncertainty}} = \pm 0.256 \text{ sec}$$

This shows that the fuel-used estimate from a GRO tank during an ascent maneuver has a 3σ uncertainty of ± 1.1 lbm. Thus, computing that the total fuel used during the ascent from that tank is 65.8 lbm, the relative error in the fuel-remaining estimation is 1.7 percent. Note that this method is an approximation that relies on engine performance to follow the curves obtained during ground testing.

Method 3: Conceptual Model of the Propulsion System

A conceptual model of the propulsion system involves creating a schematic representing the layout of the propellant piping, tank, and thruster configuration. Then a set of mathematical expressions must be developed to describe the physics of this system, using the data obtained from the manufacturers on such system characteristics as the flow resistance through the piping, characteristic propellant velocity and thrust coefficients for all thrusters, the throat areas of the thrusters, and the temperature and pressure of the tanks. A good example of the development of such a model is the GOES-I bipropellant system model.

The GOES-I propellant system consists of a pressurant tank, a fuel tank, an oxidizer tank, one main satellite thruster (MST), and 12 attitude and orbit control thrusters (AOCT) arranged in strings A and B, each containing six AOCTs.

A model representation of the bipropellant system consisting of only one thruster (e.g., the MST) may be used to derive the following set of governing equations representing the physics of the system (Figure 1):

$$P_c = P_f - \sum K_f \dot{\omega}_f^2 \quad (1-28a)$$

$$P_c = P_o - \sum K_o \dot{\omega}_o^2 \quad (1-28b)$$

$$F = A_t C_f P_c \quad (1-28c)$$

$$F = (\dot{\omega}_o + \dot{\omega}_f) I_{SP} \quad (1-28d)$$

$$F = A + B \cdot I_{SP} \quad (1-28e)$$

where

- $\sum K_o$ = line resistances (oxidizer)
- $\sum K_f$ = line resistances (fuel)
- A_t = throat area
- A, B = coefficients in Equation (1-28e)
- P_o = oxidizer tank pressure
- C_f = orifice coefficient
- P_f = fuel tank pressure

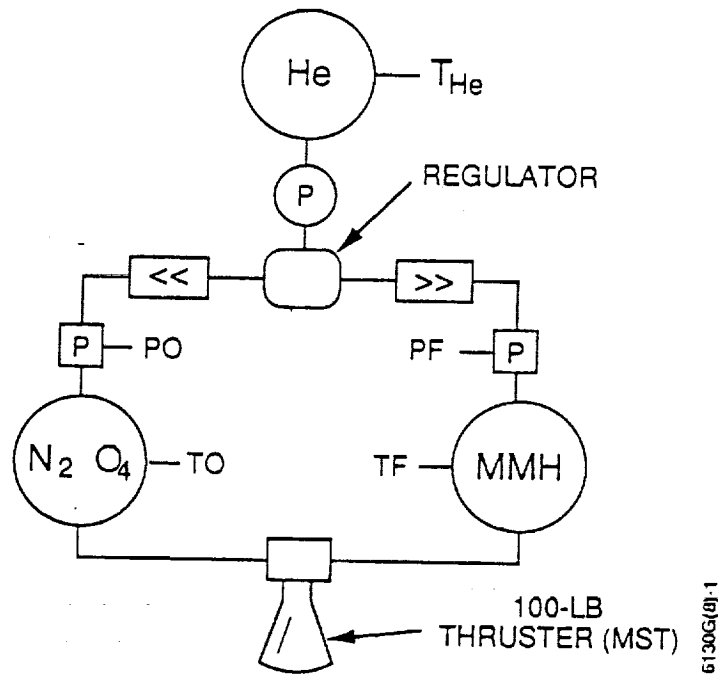


Figure 1. Schematic of the Simplified Bipropellant System Containing Only the Main Satellite Thruster

- $\dot{\omega}_f$ = fuel flow rate
- $\dot{\omega}_o$ = oxidizer flow rate
- P_c = chamber pressure
- F = thrust
- I_{sp} = specific impulse

This simple model can then be expanded to include the entire system. The conceptual model representing the system functionalities is shown in Figure 2.

Using the conceptual model in conjunction with the propellant system's physical constants, a set of governing equations relating propellant flow rates and thruster chamber pressure can be derived for each of the 13 thrusters in the same manner as for MST.

Then the mathematical representation of the functionality of the whole system is accomplished in combining the above equations for AOCTs and MST by applying a physical constraint of propellant flow continuity inherent to the system. That is, propellant mass flowing into a junction is equal to propellant mass flowing out of that junction. For example

$$\dot{\omega}_{Rt} = \dot{\omega}_f + \dot{\omega}_{fA} + \dot{\omega}_{fB} \quad (1-29)$$

Solving the system of equations described above will give the propellant flow rates through each thruster and the chamber pressure of each thruster. Propellant used due to each thruster is then the product of the flow rate and thruster on time. The propellant remaining may also be calculated by using the chamber pressure in the thrust and I_{sp} performance data curves for each thruster.

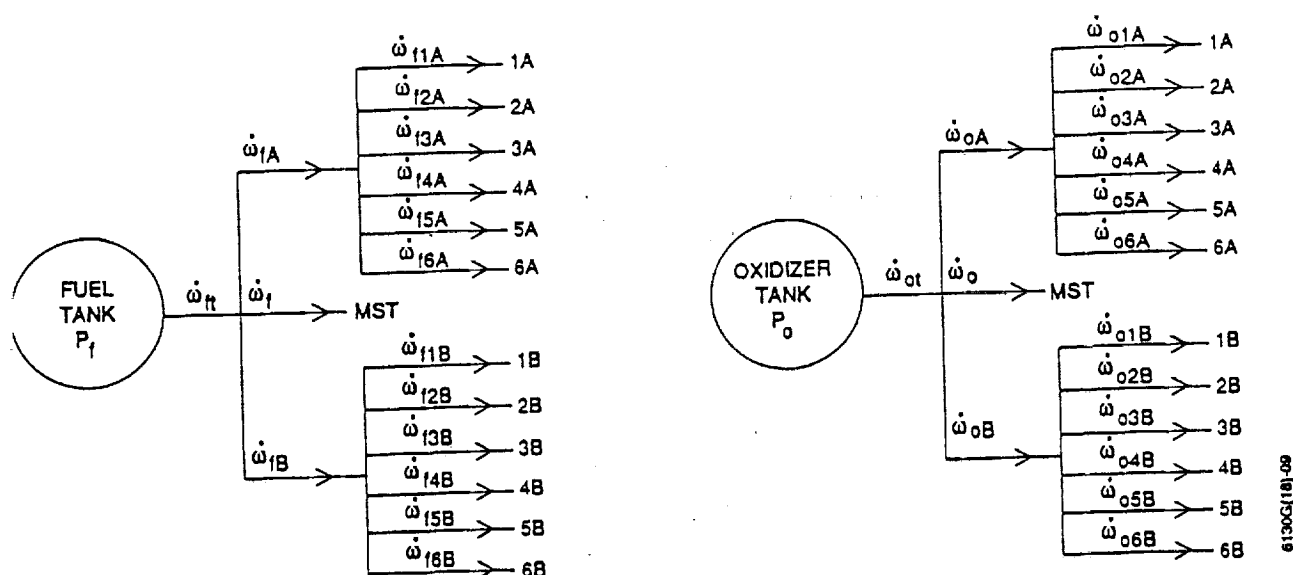


Figure 2. Conceptual Schematic of the GOES-I Propulsion System

Currently, this model is used for propellant estimation in pressure regulated mode of operation. However, by solving the system of equations for each new pressure reading in the propellant tanks, this model may be used for propellant estimation in blowdown mode.

The uncertainty in the conceptual model method comes mainly from the error in the flow resistance and thruster coefficients, tank pressure and temperature, and thruster on time. Also, there is added error in any method chosen to solve the system of nonlinear equations.

The error due to the method of solving the equations is simply the smallest tolerances of the variable under which the method converges to a solution. The error due to flow resistance and thruster coefficients, tank pressure, and temperature is determined by adding maximum error to these variables and then solving the equations to see the amount by which the solution under maximum error deviates from the nominal solution obtained by using nominal values of these variables. The error due to time uncertainty is simply the product of the time uncertainty and the computed propellant flow rate.

The following is an example of error in GOES-I MST firing in pressure regulated mode propellant-used prediction as computed by the bipropellant engine model using nominal propellant flow resistance and thrust coefficients as supplied by SS/Loral and assuming nominal tank pressures and temperatures

$$K_o = 110.713 \text{ lbf} \cdot \text{s}^2/\text{lbm} \cdot \text{in}^5 \quad K_f = 190.944 \text{ lbf} \cdot \text{s}^2/\text{lbm} \cdot \text{in}^5$$

$$C_F = 1.865 \quad P_o = P_f = 230 \text{ psi} \quad T_o = T_f = 21.3^\circ \text{ C}$$

and based on the following uncertainties: uncertainty in fuel and oxidizer tank pressures due to transducer accuracy and telemetry resolution (total pressure uncertainty is root-mean-squared of these two); uncertainty in temperatures due to telemetry resolution,

$$\begin{aligned} P_{\text{uncertainty(tele)}} &= \pm 0.4 \text{ psi} & T_{\text{uncertainty}} &= \pm 1.0^\circ \text{ K} \\ P_{\text{uncertainty(trans)}} &= \pm 1.35 \text{ psi} & P_{\text{uncertainty(tot)}} &= \pm 1.41 \text{ psi} \end{aligned}$$

and uncertainty in MST propellant flow resistance (K_o and K_f) and thrust (C_F) coefficients as given by SS/Loral (Reference 1)

$$\begin{aligned} K_{o\text{uncertainty}} &= \pm 0.541 \text{ lbf} \cdot \text{s}^2/\text{lbm} \cdot \text{in}^5 \\ K_{f\text{uncertainty}} &= \pm 2.583 \text{ lbf} \cdot \text{s}^2/\text{lbm} \cdot \text{in}^5 \\ C_{F\text{uncertainty}} &= \pm 0.00236 \end{aligned}$$

The resulting fuel and oxidizer flow rates and 1σ errors in the flow rates due to the above uncertainties combined with 0.00001 convergence tolerance of the flow rates when solved for using Runge-Kutta method are

$$\begin{aligned} \dot{\omega}_o &= 0.21857 \text{ lbm/s} & \text{Error}_{\dot{\omega}_o} &= 0.00090 \text{ lbm/s} \\ \dot{\omega}_f &= 0.13566 \text{ lbm/s} & \text{Error}_{\dot{\omega}_f} &= 0.00087 \text{ lbm/s} \end{aligned}$$

The uncertainty in MST on time due to the telemetry resolution is

$$t_{\text{uncertainty}} = \pm 0.023 \text{ sec}$$

Assuming the nominal GOES-I first two apogee maneuver firings, the total MST on time is 96 minutes (5,760 sec). Then the fuel and oxidizer masses used (as computed by the bipropellant engine model) are

$$\begin{aligned} dM_o &= 0.21857 \text{ lbm/s} \cdot 5760 \text{ s} = 1258.96 \text{ lbm} \\ dM_f &= 0.13566 \text{ lbm/s} \cdot 5760 \text{ s} = 781.40 \text{ lbm} \end{aligned}$$

The errors in the propellant-used computations are:

from errors in flow rates:

$$\text{Error}_{dM_o, \dot{\omega}_o} = \pm 5.18 \text{ lbm}$$

$$\text{Error}_{dM_f, \dot{\omega}_f} = \pm 5.01 \text{ lbm}$$

from errors in thruster on time:

$$\text{Error}_{dM_o, t} = 0.005 \text{ lbm}$$

$$\text{Error}_{dM_f, t} = 0.003 \text{ lbm}$$

Thus, based on the total propellant used during the burn, the 3σ error in fuel used is ± 15.03 lbm, the 3σ error in oxidizer used is ± 15.54 lbm, the relative error in the fuel-used estimation is 1.9 percent, and the relative error in the oxidizer-used estimate is 1.2 percent.

2. CALIBRATION TECHNIQUES

The models discussed in Part 1 of this document neglect to take advantage of the actual performance data of the spacecraft during the mission, which can be determined from the actual orbit achieved after the burn or the status of the orbit during the burn. That is, actual thrust delivered by the engines can be deduced from the orbital data available through tracking. This section presents a method of calibrating the conceptual propellant system model by using the actual thrust of the GOES-I satellite as determined from the orbit achieved.

Biprop differential corrector is a FORTRAN program that modifies the parameters of the GOES Bipropellant engine model (developed using the algorithm described in Part 1; see Reference 2) until the solution for total thrust obtained by the model matches the observed total thrust produced by the GOES propulsion system. The correction applied to the parameters is based on the information, according to SS/Loral, that the engine components most likely to vary during a burn are the propellant flow resistance coefficients for the MST section of the piping. Since the propellant piping is such that there are no isolated thrusters, the AOCTs are also affected by the varying MST resistance coefficients. However, the AOCTs are not affected when the MST is off, because MST off indicates zero propellant flow to the piping with varying resistance coefficients. Thus, Biprop differential corrector is used only when the MST is on—that is, during the NASA phase of the mission. In summary, the Biprop differential corrector is designed to correct for total thrust produced by the MST and the AOCTs combinations by adjusting the propellant (both oxidizer and fuel) flow resistance coefficients of the MST piping. The single constraint on varying the MST fuel and oxidizer resistance coefficients, given by SS/Loral, is that the mixture ratio (the ratio of the oxidizer flow rate and the fuel flow rate) for the MST must equal a predetermined constant. This section discusses (1) calculation of total thrust and average MST mixture ratio taking into account the AOCT duty cycles; (2) differential corrector requirements; (3) the differential corrector algorithm; and (4) some examples to illustrate the function and performance of the Biprop differential corrector program.

2.1 CALCULATION OF TOTAL AVERAGE THRUST AND AVERAGE MST MIXTURE RATIO

During a burn, the AOCTs are usually fired for a shorter time period than the MST. The on time of the AOCTs is described by a duty cycle (percentage of the burn time that the AOCTs are on). The MST stays on for the entire burn period. The equation for the total thrust is the average of all thrusters that are firing weighted according to each thruster's on time. As was shown in the study of the effects of multiple thruster firing on thruster performance (see Reference 3), for the total thrust magnitude calculations it is valid to assume an average duty cycle for all the AOCTs that are on. Likewise, it is valid to assume that all AOCTs that are on are firing at the same time and at the beginning of the burn. Therefore, the equation for total weighted average thrust is a sum of two parts: one for the MST firing alone and another for the MST firing together with the AOCTs. Hence,

$$T_{av} = T_{MST(off)} \cdot (1 - \text{Duty Cycle}/100) + (T_{MST(on)} + \sum T_{AOC}) \cdot (\text{Duty Cycle}/100) \quad (2-1)$$

where T_{av} = total weighted average thrust
 $T_{MST(off)}$ = MST thrust while AOCTs are off
 $T_{MST(on)}$ = MST thrust while AOCTs are on
 $\sum T_{AOC}$ = total thrust produced by AOCTs
Duty Cycle = percent of the burn time that the AOCTs are on

The mixture ratio used in the differential corrector must also be averaged, taking into account the duty cycles. The mixture ratio in the MST changes when the AOCTs go on, because the flow rates to the MST are changed. Therefore, the average MST mixture ratio must be calculated in a fashion similar to average, total thrust calculations. The formula for the weighted average MST mixture ratio is

$$MR_{av} = MR_{MST(off)} \cdot (1 - \text{Duty Cycle}/100) + MR_{MST(on)} \cdot (\text{Duty Cycle}/100) \quad (2-2)$$

where MR_{av} = average weighted MST mixture ratio
 $MR_{MST(off)}$ = MST mixture ratio while AOCTs are off
 $MR_{MST(on)}$ = MST mixture ratio while AOCTs are on

This average mixture ratio is constrained to equal a predetermined value.

2.2 DIFFERENTIAL CORRECTOR REQUIREMENTS

The differential corrector algorithm has two requirements for the function that describes the system:

1. The function must be continuous over a chosen interval.
2. The function must be differentiable on this interval.

Both of these requirements must be true for total thrust and MST mixture ratio as functions of the MST flow resistance coefficients. Since the system being modeled is a physical system, the thrust produced by the system must be directly related to the propellant flow in the system. From the governing equations of the bipropellant engine model (see Reference 4) we have for any given thruster

$$P_c = P_f - \sum (K_{f(i)} \cdot \dot{\omega}_{f(i)}^2) \quad (2-3)$$

$$P_c = P_o - \sum (K_{o(i)} \cdot \dot{\omega}_{o(i)}^2) \quad (2-4)$$

$$T = (A_t \cdot C_f) \cdot P_c \quad (2-5)$$

where P_c = chamber pressure
 P_f = fuel pressure
 P_o = oxidizer pressure
 K_f = fuel resistance coefficient
 K_o = oxidizer resistance coefficient
 $\dot{\omega}_f$ = fuel flow rate
 $\dot{\omega}_o$ = oxidizer flow rate
 T = thrust
 A_t = throat area of the thruster
 C_f = thrust coefficient

Therefore, from Equations (2-3) and (2-4), $P_c(K_f)$ and $P_c(K_o)$ are linear functions. Since T is proportional to P_c , $T(K_f)$ and $T(K_o)$ are also linear functions. Thus, taking $K_{f(MST)}$ and $K_{o(MST)}$ to be variable resistances for the MST, the functions that relate these resistances to the total thrust, $T(K_{f(MST)})$ and $T(K_{o(MST)})$, are linear by the above argument, and thereby meet the requirements of the differential corrector algorithm.

Intuitively, there must be a smooth relationship between the flow resistance coefficients and the flow rates. That is, the flow rate of a propellant in a pipe is smoothly related to the resistance of the pipe's interior surface. The mixture ratio is simply the oxidizer flow rate divided by the fuel flow rate. Also, the flow rates are never zero, since the MST is always on when the differential corrector is required. Therefore, the mixture ratio is a smooth function of the oxidizer and fuel resistance coefficients, and thus meets the requirements of the differential corrector algorithm.

2.3 DIFFERENTIAL CORRECTOR ALGORITHM

The problem of thrust correction is defined by two variables and two constraints. The two variables are the fuel resistance coefficient and the oxidizer resistance coefficient. The two constraints are that the computed thrust must equal the actual thrust and that the computed mixture ratio must equal the actual mixture ratio. The requirement that mixture ratio be fixed implies that the actual mixture ratio is equal to the nominal mixture ratio within a specified tolerance. The following procedure must be used in performing the differential correction on the bipropellant engine model:

1. Obtain the actual mixture ratio (MR_a) using bipropellant engine model with nominal flow resistance coefficients supplied by the manufacturer.
2. Obtain the actual, total thrust magnitude from the calibrated maneuver mode.
3. Until the computed total thrust (T_n) and the computed mixture ratio (MR_n) are within the specified tolerance of the constraints, iterate with

$$K_{f(MST)}(i+1) = K_{f(MST)}(i) + \Delta K_{f(MST)} \quad (2-6)$$

$$K_{o(MST)}(i+1) = K_{o(MST)}(i) + \Delta K_{o(MST)} \quad (2-7)$$

where $\Delta K_{f(MST)}$ and $\Delta K_{o(MST)}$ are obtained via the differential corrector method

$$\begin{bmatrix} \Delta K_{o(MST)} \\ \Delta K_{f(MST)} \end{bmatrix} = \begin{bmatrix} \delta T / \delta K_{o(MST)} & \delta T / \delta K_{f(MST)} \\ \delta MR / \delta K_{o(MST)} & \delta MR / \delta K_{f(MST)} \end{bmatrix}^{-1} \begin{bmatrix} T_a - T_n \\ MR_a - MR_n \end{bmatrix} \quad (2-8)$$

where $\delta K_{o(MST)}$, $\delta K_{f(MST)}$ = perturbation applied to the coefficients

δT , δMR = $(T_{pert} - T_n)$, $(MR_{pert} - MR_n)$ respectively

T_{pert} , MR_{pert} = thrust and mixture ratio, respectively, computed by BIPROP using perturbed coefficients $K_{f(MST)}(i) + \delta K_{f(MST)}$, $K_{o(MST)}(i) + \delta K_{o(MST)}$

T_n , MR_n = thrust and mixture ratio, respectively, computed by BIPROP using unperturbed coefficients

2.4 EXAMPLES OF BIPROP DIFFERENTIAL CORRECTOR

Table 1 gives some examples of the differential corrector performance. The numbers used for the actual thrust (T_a) were chosen only for the testing purposes. The nominal resistance coefficients were part of the data given by the manufacturer for testing the bipropellant engine model software. As shown in the table, the differential correction makes good progress in only two to three iterations with relative error for thrust and mixture ratio specified at under 0.5 percent. Note that in Case 2 we find significantly higher resistance than in Case 1, although the difference between actual and total thrust in both cases is almost the same. This may be understood as follows:

Let

$$DT_{MST} = T_{MST(off)} - T_{MST(on)} \quad (2-9)$$

so the DT_{MST} is the change in the MST thrust caused by AOCTs firing. Then, substituting DT_{MST} into Equation (2-1), we get

$$T_{av} = T_{MST(off)} + (\text{Duty Cycle}/100) \cdot \left(\sum T_{AOC} - DT_{MST} \right) \quad (2-10)$$

Table 1. Differential Corrector Performance

C A S E	Thrusters Firing	Nominal resistance coefficients as supplied by FACC		Nominal total thrust computed by BIPROP	Actual average mixture ratio computed by BIPROP	Actual total thrust from calibrated maneuver mode	Actual resistance coefficients computed by Differential Corrector to obtain actual thrust		Total number of iterations performed by Differential Corrector to obtain the actual thrust
		Oxidizer ($K_{o(MST)}$)	Fuel ($K_{f(MST)}$)				Oxidizer ($K_{o(MST)}$)	Fuel ($K_{f(MST)}$)	
#	names			(T_n)	(MR_a)	(T_a)			#
1	MST only	110.713	190.944	108.876	1.611	93.6	176.079	297.618	3
2	MST, 2A	110.713	190.944	110.748	1.644	95.0	178.624	303.305	2
3	MST, west face AOCs	110.713	190.944	112.613	1.643	102.0	141.416	255.606	2
4	MST, all AOCs	110.713	190.944	116.034	1.642	140.0	53.046	95.709	3

NOTE: For the examples here, the differential corrector uses the following inputs:

Specified Relative Error for T_a and MR_a : 0.5%

Duty Cycle: 10%

Burn Time: 3,000 sec

Although $DT_{MST} > 0$, we observe that $(\sum T_{AOC} - DT_{MST}) > 0$. Also, from Equations (2-3), (2-4), and (2-5), we know that $T_{MST(off)}$ decreases with increasing resistances. Likewise, from Equation (2-10), we have

$$T_{MST(off)} = T_{av} - (\text{Duty Cycle}/100) \cdot (\sum T_{AOC} - DT_{MST}) \quad (2-11)$$

This shows $T_{MST(off)} < T_{av}$ in Case 2, while $T_{MST(off)} = T_{av}$ in Case 1. Therefore, we expect Case 2 to require higher resistances. Moreover, as the resistances increase, the flow rates to the MST decrease; and, from continuity conditions, the flow rates to AOCTs must increase. Thus,

$(\text{Duty Cycle}/100) \cdot (\sum T_{AOC} - DT_{MST})$ increases as resistances increase. Similarly,

$(\text{Duty Cycle}/100) \cdot (\sum T_{AOC} - DT_{MST})$ increases as the number of the AOCTs firing or the Duty Cycle increases. That is, the increase in required resistances to obtain the same average thrust becomes more marked as the number, or the duty cycle, of the AOCTs firing with the MST increases.

Case 4 shows that the resistance coefficients may also be decreased in order to account for a better thruster performance than expected.

When such a calibration technique is used, the propellant flow rates and the chamber pressure of each thruster that was firing during the burn are adjusted in the process to reflect more closely the actual performance of the thrusters. Using the information of case 1 from Table 1 as an example, if the actual thruster performance is 86 percent of the predicted performance then the difference in fuel used computed using nominal and calibrated flow resistance coefficients is 107 lbm. For GOES-I 107 lbm is equivalent to 2.5 years of mission lifetime. Therefore, the propellant-remaining calculations derived by the conceptual model of the system are more realistic, since the model reflects the actual performance of the propellant system as observed during the mission.

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